DISCUSSION

An historic theorem in plane geometry. I was much interested in Capt. Robert A. Laird's demonstration, page 361 of the October MATHEMATICS TEACHER. It is truly an interesting theorem; but it can be established very much more briefly as follows:

Dividing equation (1) by equations (2) and (3) and clearing of fractions we obtain the equation,

$$BD:AE:CF = AD:BF:CE.$$

Therefore A, B and C are in a straight line because they are points on the sides (produced) of the triangle DEF, and by the converse of a well known theorem of Menelaus the points are collinear. (If three points are located on the sides of a triangle so that the product of any three segments that do not have a common extremity equals the product of any other three such segments the points are collinear. (See art. 105 Lachlan *Modern Pure Geometry.*)

Similarly it is easily demonstrated that two intersections of the common internal tangents and one intersection of common external tangents are collinear, the six points on three straight lines. Also that the lines through the centers of the circles and the intersections of the internal tangents are concurrent.

W. H. CARNAHAN,
High School, Washington, Ind.

Geometry Speaks. All my life I have suffered from inability to make myself acceptable in the younger set where my brother Arithmetic has been at home for many years. Sister Algebra, in her day, experienced somewhat the same difficulty, but after Arithmetic had taken her to school with him several times in the eighth and ninth grades, people seemed to notice a family resemblance, and while some remained only on speaking terms with her, there were a good many boys and girls between whom and herself a pleasant attachment grew up.

She was really more attractive in personal appearance. Her $\sqrt{\ }$ collars were neat and easy to copy and generally be-

coming, and when it came to writing examples, 6 apples and 10 beans and 5 carrots added to 4 apples and 3 beans and 7 carrots looked more clumsy than

$$\begin{array}{r} 6a + 10b + 5c \\ \underline{4a + 3b + 7c} \end{array}$$

so that, gradually, she grew to have a place of her own in the list of studies and to become something of a favorite with both teachers and pupils.

Unfortunately I have always seemed to be the oldest and most dignified of our trio. Whether this is due to the matter of association with upper classmen or to a certain formal manner which I have felt obliged to assume in order to keep up the family reputation is uncertain. Whatever the cause, I have always found none too easy the process of introduction to *Senior* High School pupils, even, and have made their acquaintance under heavy odds.

It may be imagined, then, that the prospect of being brought face to face, at different times during the year, with youngsters *just entering their teens* was somewhat appalling. However, all studies are at the mercy of Educators and if Educators say that I must enter the Junior High School into the Junior High School I go—and, having gone, I find what threatened to be an unhappy experience developing more and more into a pleasure.

As a family we three, Arithmetic, Algebra and Geometry, have always lived in wonderful harmony, because, of course, we stand for the same ideals—fidelity to the truth and willingness to see other than our points of view, but, I am bound to confess, we have never worked so effectively as since we all went into business together, so to speak. Everyone knows that we are of the multiform family and have the power of simultaneous existence in widely separated districts. For example, I may, at the same moment, be meeting a class in San Francisco and one in Boston, and we are presented in as many different fashions as there are textbooks and teachers. We enter all classes of intellectual society and Algebra and I are often more pleasantly received than Arithmetic has been. Children sometimes tire of

working with him, year after year, even though those who are fond of him find that their liking increases with length of their acquaintance.

A recent experience with a ninth grade division of children who had not taken very kindly to Arithmetic has interested me and set me to wondering if there may not be some way of making myself sufficiently adjustable and sympathetic to meet pupils a year or even two years younger than these and win them over. There were twenty-seven pupils in this class which, early in September, met for a first lesson in General Mathematics. For two or three weeks I just looked on at recitations while they found out how many dollars (t) apples cost at (f) cents each and what X^2 meant if X meant 5, and how to express the sum of twice A and three times B and what that sum would be if A happened to stand for 7 and B for 3, and how Algebra would tell them the depth of sand in a box, four and a half feet by eight, if a cubic yard of sand was dumped in and leveled off, and matters of that sort.

At first they seemed terribly stupid and I thought the teacher must get tired of saying, "If you had 4 apples and each one cost 5 cents how much would they all cost? Yes. Now, if you had 10 apples, etc. Now, if you had t apples and each one cost f cents how much would they all cost?" She never seemed tired, perhaps because the children acted so differently. Some were cautious, some very venturesome and ready to give any answer, and some so timid that they seemed afraid to speak at all. These timid ones hardly ever had to answer alone until the bold ones had made mistakes and corrected them and the whole class had answered in concert. (I heard the teacher say *that* practice wansn't "according to Hoyle" but children were made before theories, and some men became brave because of the shouts of others) I didn't know what she meant and I didn't care, after a while, because I grew so anxious for the slow ones to get brave enough so that I wouldn't scare them when I came on the scene.

I needn't have worried! They learned my addition, subtraction, multiplication and division axioms in a fortnight, while they were doing some of Algebra's equations, more read-

ily, I thought, than some of the grown-ups learn them. One day, near the end of a period, the teacher began walking around and looking at the children's feet. They were all interested, watching and wondering, until she said, suddenly, "I was looking for a shining pair of shoes. Marion, yours look well. Come out before the class and see if you can place your feet so that they will make an acute angle with each other. Acute means, Class?" "Pointed." "Thin." "Sharp." "Less than a right angle?" were some of the answers. When Marion's feet made an angle Helen was told to put two rulers on the floor, meeting just inside the heels of Marion's boots. Then Alice drew, on the board, two lines making about the same angle which was agreed to be 45 degrees.

There wasn't time for any more, that day, and the teacher didn't say anything about remembering angles, but, a day or two later, she drew a queer looking thing on the board and asked the children if they knew what it was. Some said, "Cape Cod" and she said it was a great comfort to have it recognized because her geography teacher once told her that she would never be able to draw a map so that anyone would know what it represented. Then she drew something to stand for a boat with a man and a boy in it, ran a north and south line through the boat and asked if some one would draw a line meeting that one at an angle of 45 degrees. Half the class wanted to do it, and the lesson stopped after it was done.

Later, in odd bits of time, the children learned to construct a perpendicular, to make an angle equal to a given angle, how to tell and to draw, complementary and supplementary angles and how to select alternate interior and alternate exterior angles. They weren't told many things but allowed to do what they thought right, and if they made mistakes, almost always some one in the class could correct them.

I was particularly interested in a white-faced, mouselike little boy who didn't do anything except stand when the teacher told him to and repeat after her what she said, until it came to constructing equal angles. He got that, right away, and was always the first one called on if company came in. From the time he learned to do this he plucked up his courage and now he

wants to go to the board to try out work in Algebra although, at first, he wasn't on good terms with her, at all.

I'm a member of the General Mathematics Class now, in good and regular standing. The children have learned the four fundamental processes in Algebra, they can multiply binomials by inspection and even factor a little—though they don't know it—they can reduce some fractions to lowest terms, they can make up simple angle problems and solve them and they are almost ready for triangles. A boy in the sloyd class has made a big wooden protractor and, pretty soon, they are going to find the height of the room by "magic" and test the result by measurement.

I have made a long speech and I have made it for this reason. Dressed in my academic costume and with my dignified demeanor on I am a fit associate for people getting ready for college, but, dressed in my play clothes and allowed to have a good time with Junior High School youngsters I think I can both give them pleasure and fit them, without their suspecting it, for real work, later on. Why not try letting me play with them and see if I am not right?

EVA M. PALMER,
High School, Winchester, Mass.

Notes on the Teaching of Arithmetic. For years we have given the child special help and devices in reading. He has had his families of words, his phonics, his charts, and illustrated books. In arithmetic what have we done? Tried the topical plan, the spiral plan, and other plans. Given some games and drills and helps to make the work more attractive, but never has the real cause for failure been reached. We have done nothing to remove the drudgery attendant on acquiring the mechanics of the subject. The natural combination of numbers, the aids to computation, the beauties of the number system have been kept away from the pupil and from many teachers. Why not make a study of arithmetic?

To be successful in arithmetic get away from drudgery, avoid too cumbersome computing, study the number system, make arithmetic a joy. You can't get speed and accuracy with long computations. And you can't get problem work until the pupil has facility in number work.

Our number system is so beautiful, so easy, so full of possibilities that if only a small amount of time is given to it your number work will improve.

You must consider—the order in which number is taught,
—the composition, decimal and factor.

Begin with doubles: $1 + 1$, $2 + 2$, etc.

Then use doubles plus 1, plus 2, etc.

An attractive chart has been developed for this work.
$2 + 2 = 4$, then $2 + 2 + 1 = 5$, or $2 + 3 = 5$.
Use the decades: $2 + 2$, $12 + 2$, $22 + 2$,
also $12 = 10 + 2$.

Emphasize this continually. It helps all along the line.
$15 \times 12 = 15(10 + 2)$. Introduce some areas. (Oral).

Add beginning at the left: 25, 16, 41, 17, 20, 30, 70, 80, 85, 91, 92, 99.

Change the order of teaching tables. There is no reason why they should be taught in consecutive order. After some preliminary work, teach 5,s, 10's, and 20's. Your pupil knows many of the facts. The grocery store and candy counter have been his teachers. Incidentally the boy feels pretty big when he can handle 20's, and unconsciously he learns his 2's.

Then teach 2's. 4's and 8's naturally follow. Reverse each pair of factors. Teach 1/4, 1/2, 1/8. Use Dry Measure for your application. This table is close to the pupil's experience. Reversing factors teaches other tables.

Next teach 3's, 6's, 12's. Teach 1/3, 1/6.

9 is a hard table for many children. It is all done now except 7×9 and 9×9. Teach 9's.

Use the yard for concrete work in 3's, 6's, and 9's, 12's.

Teach 7's. This is all done except 7×7 and 7×11. Use the week for application.

Teach 11's. This is all complete except 11×11.

Don't be afraid to drill. Lessen your work as teacher and increase the efficiency of your pupil by giving keyed examples, no two alike. This multiplies the drill work in fundamental processes and prevents copying. Key all examples which you give

for speed and accuracy. Such speed work applies to Addition, Subtraction, Multiplication, Division, Mixed Numbers, Decimals, at board or seat.

Solve:	864
1680175÷321	941
	582

These are keyed by two digits. You can use other keyed schemes also. The teacher can make such examples or purchase them.

To check division, add the subtrahends that have been checked.

Teach squares and cubes.

Teach the relations of odd and even numbers in series. There are many interesting laws connected therewith.

Make use of factors constantly. Express a solution, delaying computation as long as possible.

Make use of architects' triangles. Take advantage of $\sqrt{2}$ and $\sqrt{3}$.

Use factors in reduction of fractions, even in decimals. Make use of the fact that the denominater of a decimal fraction is a power of 10.

Apply decimals to interest. Use no other method.

By the above means, so briefly outlined, our pupils will learn not only to compute but to think. Their high school mathematics will no longer suffer.

R. L. Short.
St. Louis, Mo.